Eur. Phys. J. A **9**, 269–276 (2000)

THE EUROPEAN PHYSICAL JOURNAL A

Pion and kaon electromagnetic form factors in a $\mathrm{SU_L(3)} \otimes \mathrm{SU_R(3)}$ effective Lagrangian

Yun Chang Shin[1], Bong Soo Han[1], Myung Ki Cheoun[1,2,a], K.S. Kim[1], and Il-Tong Cheon[1]

[1] Department of Physics, Yonsei University, Seoul, 120-749, Korea
[2] IUCNSF, Seoul National University, 151-742 Korea

Received: 20 December 1999 / Revised version: 19 August 2000
Communicated by A. Schäfer

Abstract. A $SU(2)$ effective Lagrangian is extended to a $SU_{\mathrm{L}}(3) \otimes SU_{\mathrm{R}}(3)$ by including the vector and axial vector meson. With this effective Lagrangian, electromagnetic form factors of charged pion and kaon are calculated in both time- and space-like regions. The pseudoscalar meson loops are taken into account. Good agreement with experimental data is obtained for those form factors and charged pseudoscalar meson radii. Decay widths of $\rho \to \pi\pi$ and $\phi \to K^+K^-$ are also calculated and shown to agree with experimental data very well.

PACS. 12.39.Fe Chiral Lagrangians – 12.40.Vv Vector-meson dominance – 13.40.Gp Electromagnetic form factors – 14.40.Ev Other strange mesons

1 Introduction

At energy below 1 GeV, the vector meson plays an important role in electromagnetic interactions of the hadron. The vector meson dominance model (VMD) has been proved remarkably successful in the description of electromagnetic form factors and decays of the hadron, although it is a phenomenological approach. Many approaches, such as the hidden gauge symmetry approach (HGS) [1], the massive Yang-Mills approach (MYM) [2], and so on, have been developed to include the vector meson in a fundamental manner. By taking higher-order terms into account, redefining suitable field and adjusting parameters, all of the model can be shown to be equivalent [3]. However, a simple addition of higher-order terms is not a convenient method for those calculations. In our previous paper [4], we have proposed an effective chiral Lagrangian for the description of vector and axial-vector mesons by considering all the relevant symmetries and the low-energy constraints from chiral perturbation theory (ChPT). In that paper, relevant experimental data are reproduced with only mass terms and kinetic terms of spin-1 meson fields. The spin-1 mesons are introduced in the nonlinear realization of chiral symmetry, with which it is easy to check consistency with chiral perturbation theory. In constructing our model Lagrangian, we have stressed simplicity. Only mass terms and kinetic terms of spin-1 meson fields are necessary to meet experimental results. In some approach, such as $O(p^4)$ expansion of ChPT, $\mathcal{L}_2$ Lagrangian gives loop contribution as well known [5], which helps a good phenomenological description. But our effective Lagrangian theory, which is aimed for large energy process, uses $O(p^2)$ expansion because most of the higher-order contributions in other approaches are incorporated by a single change in the kinetic terms of vector field with only one parameter in our model. Since full reviews concerning effective theories and their relationships to other approaches can be found in other papers [2,3], we skip them here.

In this paper our previous Lagrangian, with a brief summary, is extended to $SU(3)$ in section 2. In section 3, the electromagnetic pion and kaon form factors and some related decays with this Lagrangian are presented with detailed discussions. A brief summary is done in the final section.

2 Lagrangian

Our Lagrangian consists of a pseudoscalar meson sector $\mathcal{L}(\pi)$, a spin-1 vector and axial vector meson sector $\mathcal{L}(V, A)$, and a term of interactions with scalar particles $\mathcal{L}_{\mathrm{s}}$, which comes from mass splittings in $SU(3)$ extension, *i.e.*,

$$\mathcal{L} = \mathcal{L}(\pi) + \mathcal{L}(V, A) + \mathcal{L}_{\mathrm{s}}. \tag{1}$$

The Lagrangian for the pseudoscalar meson sector, which is a leading Lagrangian of the ChPT, is given as

$$\mathcal{L}(\pi) = \frac{f^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle + \frac{f^2}{4}\langle U^\dagger \chi + \chi^\dagger U\rangle \ , \tag{2}$$

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \ , \tag{3}$$

[a] e-mail: cheoun@phya.snu.ac.kr

where bracket denotes a trace in flavor space, f is a pseudoscalar meson decay constant, chiral field $U = \exp(i2\pi/f)$ with $\pi = T^a\pi^a$, $T^a = \lambda^a/2$ $(a = 1, 2, ...8)$. External gauge fields are introduced via v_μ and a_μ. The χ is defined by $\chi = 2B_0(\mathcal{S} + i\mathcal{P})$. Explicit chiral symmetry breaking due to current quark masses can be introduced by treating those masses as if they were uniform external scalar field S [3].

Under a local $SU(N_f)\times SU(N_f)$ gauge transformation, $U \to g_\mathrm{R} U g_\mathrm{L}^\dagger$, χ and D_μ transform as U does. The above Lagrangian is invariant, provided that the external gauge fields transform as

$$
\begin{aligned}
&v_\mu + a_\mu \to g_\mathrm{R}(v_\mu + a_\mu)g_\mathrm{R}^\dagger - i\partial_\mu g_\mathrm{R}\cdot g_\mathrm{R}^\dagger ,\\
&v_\mu - a_\mu \to g_\mathrm{L}(v_\mu - a_\mu)g_\mathrm{L}^\dagger - i\partial_\mu g_\mathrm{L}\cdot g_\mathrm{L}^\dagger ,\\
&\mathcal{S} + i\mathcal{P} \to g_\mathrm{R}(\mathcal{S} + i\mathcal{P})g_\mathrm{L}^\dagger .
\end{aligned}
\tag{4}
$$

The non-linear realization of chiral symmetry is expressed in terms of $u = \sqrt{U}$ and $h = h(u, g_\mathrm{R}, g_\mathrm{L})$ defined from $u \to g_\mathrm{R} u h^\dagger = h u g_\mathrm{L}^\dagger$. In this realization, we naturally have the following covariant quantities:

$$
\begin{aligned}
i\Gamma_\mu &= \frac{i}{2}(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger)\\
&\quad + \frac{1}{2}u^\dagger(v_\mu + a_\mu)u + \frac{1}{2}u(v_\mu - a_\mu)u^\dagger ,\\
i\Delta_\mu &= \frac{i}{2}(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger)\\
&\quad + \frac{1}{2}u^\dagger(v_\mu + a_\mu)u - \frac{1}{2}u(v_\mu - a_\mu)u^\dagger ,\\
\chi_+ &= u^\dagger\chi u^\dagger + u\chi u ,
\end{aligned}
\tag{5}
$$

whose transformations are carried out in terms of h, *i.e.*, $\Gamma_\mu \to h\Gamma_\mu h^\dagger - \partial_\mu h\cdot h^\dagger$, $\Delta_\mu \to h\Delta_\mu h^\dagger$, and $\chi_+ \to h\chi_+ h^\dagger$. With these quantities, the Lagrangian in eq. (2) can be rewritten as

$$
\mathcal{L}(\pi) = f^2\langle i\Delta_\mu i\Delta^\mu\rangle + \frac{f^2}{4}\langle\chi_+\rangle. \tag{6}
$$

As for the massive spin-1 mesons, we include only the mass and kinetic terms [4]

$$
\begin{aligned}
\mathcal{L}(V, A) = m_\mathrm{V}^2\langle(V_\mu - \frac{i\Gamma_\mu}{g})^2\rangle + m_\mathrm{A}^2\langle(A_\mu - \frac{ir\Delta_\mu}{g})^2\rangle\\
-\frac{1}{2}\langle({}^G V_{\mu\nu})^2\rangle - \frac{1}{2}\langle(A_{\mu\nu})^2\rangle
\end{aligned}
\tag{7}
$$

with

$$
\begin{aligned}
{}^G V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] - iG[A_\mu, A_\nu] ,\\
A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[V_\mu, A_\nu] - ig[A_\mu, V_\nu] ,
\end{aligned}
\tag{8}
$$

where $V_\mu = T^a V_\mu^a (A_\mu = T^a A_\mu^a)$ denotes spin-1 vector (axial-vector) meson field and g denotes a $V\pi\pi$ coupling constant. The chiral transformation rules of spin-1 fields are expressed in terms of h

$$
V_\mu \to hV_\mu h^\dagger - \frac{i}{g}\partial_\mu h\cdot h^\dagger \ ,\ A_\mu \to hA_\mu h^\dagger. \tag{9}
$$

Note that we have introduced a new form of ${}^G V_{\mu\nu}$. The chiral symmetry is preserved for any value of G at chiral limit in ${}^G V_{\mu\nu}$, so that the value of G cannot be determined from the chiral symmetry. If G is equal to g as in the HGS approach, the result may reproduce experimental data by including other higher-order terms.

The $\mathcal{L}_\mathrm{s}$ term is introduced in the following way. It considers effects coming from mass splittings of strange and non-strange particles in terms of interaction Lagrangians between scalar particles and other mesons (pseudoscalar, vector and axial-vector mesons). In the presence of the interaction, scalar particle field $\mathcal{S}$ satisfies the Klein-Gordon equation $\mathcal{S}\partial_\mu\partial^\mu\mathcal{S} + M_\mathrm{s}\mathcal{S}^2 = -2\mathcal{S}J$, where $\mathcal{S}J$ is a source term for the $\mathcal{S}$-field due to the interaction. If we assume that the kinetic term of the scalar particle is small enough to be neglected because it is massive [18], and integrate out the $\mathcal{S}$-field from the generating function,

$$
Z_\mathcal{S} = \int \mathrm{d}\mathcal{S}\ \exp\Big(\int[M_\mathrm{s}(\mathcal{S} + \frac{J}{M_\mathrm{s}})^2 - \frac{J^2}{M_\mathrm{s}^2}]\Big) , \tag{10}
$$

the resulting Lagrangian is expressed as follows:

$$
\mathcal{L}_\mathrm{s} = \frac{1}{M_\mathrm{s}^2}\langle J^2\rangle = \frac{1}{M_\mathrm{s}^2}\langle J'^2 + 2J_\mathrm{vac}J' + J_\mathrm{vac}^2\rangle , \tag{11}
$$

where

$$
\begin{aligned}
J' &= (J - J_\mathrm{vac}) =\\
&\quad -\frac{M_\mathrm{s}}{4}s_m\frac{2B_0}{f^2}(\pi^2 M + M\pi^2 + 2\pi M\pi) + M_\mathrm{s} j ,\\
j &= s_d(i\Delta_\mu)^2 + s_\mathrm{V}(gV_\mu - i\Gamma_\mu)^2 + s_\mathrm{A}(gA_\mu - ir\Delta_\mu)^2\\
&\quad + s_r\{i\Delta^\mu, gA_\mu - ir\Delta_\mu\} ,\\
J_\mathrm{vac} &= M_\mathrm{s}s_m B_0 M .
\end{aligned}
\tag{12}
$$

Here M is the current quark mass matrix (we assume that masses of u and d quarks are equal), which is given as

$$
\begin{aligned}
M_a &= 2B_0(\frac{D_a}{2\sqrt{3}}\alpha + \beta),\\
\frac{D_a}{2\sqrt{3}}\alpha + \beta &= \begin{cases} \overline{m}, & \text{for } a = 1, 2, 3\\ \frac{1}{2}(\overline{m} + m_\mathrm{s}), & \text{for } a = 4, 5, 6, 7\\ \frac{1}{3}(\overline{m} + 2m_\mathrm{s}), & \text{for } a = 8 . \end{cases}
\end{aligned}
\tag{13}
$$

B_0 is a constant related to the scalar quark condensation. j stands for the interaction, on which s_d, s_V, s_A, s_r, and s_m are free parameters. It should be noted that the above Lagrangian consists of an interaction between scalar and pseudoscalar ($M_\mathrm{s}\pi M$...), scalar and vector mesons ($M_\mathrm{s} j$), and contributions from the vacuum ($M_\mathrm{s}B_0 M$). In eq. (11), we only consider the terms with double and triple field products, so that we can write down $\mathcal{L}_\mathrm{s}$ as follows:

$$
\mathcal{L}_\mathrm{s} \sim -\frac{1}{2}(\frac{s_m}{f})^2(\tilde{M})_a^2(\pi^a)^2 + \frac{1}{2}s_m M_a j^a, \tag{14}
$$

where $\tilde{M}_a^2 = \frac{1}{6}(2B_0\alpha)^2\delta_{8a} + M_a^2$.

2.1 Mixings

There are some unphysical mixings in the Lagrangian, which can be removed by field redefintion. First, let us consider a nonet mixing. For vector bosons the nonet symmetry is good more or less. To simplify the calculation, we use this symmetry *i.e.*, ω and ϕ mesons are written as [11]

$$\omega_\mu=\sqrt{\frac{2}{3}}\omega_{\mu 1}+\sqrt{\frac{1}{3}}V^8_\mu\,,\quad \phi_\mu=\sqrt{\frac{1}{3}}\omega_{\mu 1}-\sqrt{\frac{2}{3}}V^8_\mu. \qquad (15)$$

For the mixing between axial vector mesons and pion fields, we define $A^{'}_\mu$ as

$$A_\mu = A'_\mu - \frac{r}{gf}\partial_\mu\pi. \qquad (16)$$

Through this field redefinition, a new term which is not renormalizable appears in the kinetic part. Therefore, in order to keep the kinetic terms the same as before under this field redefinition, we also redefine the vector meson field as follows [7]:

$$V_\mu = V'_\mu - \frac{Gr^2}{2g^2f^2}f_{abc}\pi^b\partial_\mu\pi^c. \qquad (17)$$

Finally, we consider the mixing between the vector meson and a photon field. The photon field enters through the external vector field $v_\mu = eQA^{\rm em}_\mu$, where $Q = T^3 + \frac{Y}{2}$. The unphysical mixing related to the photon field can be removed by the following field and charge redefinitions [8, 9]:

$$\begin{aligned} V_\mu &\to V^{'}_\mu + \frac{e'}{g}QA^{\rm em'}_\mu\,,\\ A^{\rm em}_\mu &\to A^{\rm em'}_\mu\sqrt{1-\frac{e'^2}{g^2}Q^2}\,,\\ e &\to e^{'}/\sqrt{1-\frac{e'^2}{g^2}Q^2}\,. \end{aligned}$$

2.2 Effective Lagrangian

In expanding our Lagrangian, we choose only the photon, pseudoscalar meson and vector meson parts. The Lagrangian is, then, simply summarized as

$$\begin{aligned} \mathcal{L} = &\frac{1}{2}m^2_{{\rm V}a}V_\mu V^\mu\\ &+\frac{m^2_{{\rm V}a}}{2gf_a^2}(1-\frac{Gr^2}{g})f_{abc}V^a_\mu\pi^b\partial^\mu\pi^c\\ &+eQf_{abc}A^a_\mu\pi^b\partial^\mu\pi^c\\ &-\frac{1}{4}(\partial_\mu V_\nu-\partial_\nu V_\mu)^2-\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2\\ &-\frac{e}{2g}(\partial_\mu V_\nu-\partial_\nu V_\mu)(\partial^\mu A^\nu-\partial^\nu A^\mu)\\ &-\frac{1}{2}m^2_{\pi a}\pi^a\pi^a+\frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a, \end{aligned} \qquad (18)$$

where $m^2_{{\rm V}a} = g^2(f^2_{\rm V} + s_m s_v M_a)$ and V_μ and A_μ stand for the redefined fields $V^{'}_\mu$ and $A^{\rm em'}_\mu$. In order to determine pseudoscalar meson mass and decay constants, we exploit the following covariant quantities:

$$\begin{aligned} m^2_{\pi a} &= (M_a+(\frac{s_m}{f})^2\tilde{M_a}^2)/Z^2_{\pi a}\,,\quad f_a = Z_{\pi a}f\,,\\ &\text{with}\quad Z^2_{\pi a} = (1+s_m s_d\frac{M_a}{f^2})\,. \end{aligned} \qquad (19)$$

Mass splitting between non-strange particles and strange particles is generated from the interaction of these fields with a scalar field which is given by eq. (14).

2.3 Comparison with ChPT

For processes with small momentum transfer, massive degree of freedom can be integrated out leaving an effective Lagrangian of pions and external fields, which can be compared with the Lagrangian of ChPT. By this comparison, we can check consistency of our Lagrangian at low energy.

To the order we consider, we can integrate out massive degree of freedom by replacing the massive fields by their zeroth-order solutions:

$$\begin{aligned} V_\mu &\to \frac{1}{g}i\Gamma_\mu\,,\\ A_\mu &\to \frac{r}{g}i\Delta_\mu\,. \end{aligned} \qquad (20)$$

Then the resulting effective Lagrangian has the form

$$\begin{aligned} \mathcal{L} &= \mathcal{L}(\pi)\\ &+\frac{1}{2g^2}\langle(\Gamma_{\mu\nu}+r^2G[\Delta_\mu,\Delta_\nu])^2\rangle+\frac{r^2}{2g^2}\langle\Delta^2_{\mu\nu}\rangle \end{aligned} \qquad (21)$$

with

$$\begin{aligned} \Delta^{\mu\nu} &= \partial^\mu\Delta^\nu-\partial^\nu\Delta^\mu+[\Gamma^\mu,\Delta^\nu]-[\Gamma^\nu,\Delta^\mu]\\ &= -\frac{i}{2}(\xi^\dagger F^{\mu\nu}_{\rm R}\xi-\xi F^{\mu\nu}_{\rm L}\xi^\dagger)\,,\\ \Gamma^{\mu\nu} &= \partial^\mu\Gamma^\nu-\partial^\nu\Gamma^\mu+[\Gamma^\mu,\Gamma^\nu]\\ &= -[\Delta^\mu,\Delta^\nu]-\frac{i}{2}(\xi^\dagger F^{\mu\nu}_{\rm R}\xi-\xi F^{\mu\nu}_{\rm L}\xi^\dagger) \end{aligned} \qquad (22)$$

and

$$\begin{aligned} F^{\mu\nu}_{\rm L,R} &= \partial^\mu(v^\nu\mp a^\nu)-\partial^\nu(v^\mu\mp a^\mu)\\ &\quad -i[v^\mu\mp a^\mu, v^\nu\mp a^\nu]\,. \end{aligned} \qquad (23)$$

For easy comparison, we list the contributions to the coefficient of $\mathcal{L}_4$:

$$\begin{aligned} &L^{\rm V}_1 = \frac{1}{32g^2}(1-Gr^2)^2, L^{\rm V}_2 = 2L^{\rm V}_1, L^{\rm V}_3 = -6L^{\rm V}_1,\\ &L^{\rm V}_9 = \frac{1}{4g^2}(1-Gr^2)^2, L^{\rm V}_{10} = -\frac{1}{4g^2}, H^{\rm V}_1 = \frac{1}{2}L^{\rm V}_{10},\\ &L^{\rm A}_{10} = \frac{r^2}{4g^2}, H^{\rm A}_1 = -\frac{1}{2}L^{\rm A}_{10}, \end{aligned} \qquad (24)$$

which are equivalent with Ecker *et al.*'s expressions (eqs. (5.6) and (5.9) of ref. [6]) with the following correspondence of parameters between the two formulas:

$$g \longleftrightarrow \frac{M_\mathrm{V}}{F_\mathrm{V}},$$
$$Gr^2 \longleftrightarrow \frac{F_\mathrm{V}-2G_\mathrm{V}}{F_\mathrm{V}},$$
$$r \longleftrightarrow \frac{F_\mathrm{A}}{F_\mathrm{V}}\frac{M_\mathrm{V}}{M_\mathrm{A}}, \tag{25}$$

where $F_\mathrm{V}, G_\mathrm{V}, F_\mathrm{A}, M_\mathrm{V} \simeq m_\mathrm{V}$ and $M_\mathrm{A} \simeq m_\mathrm{A}$ are the parameters introduced by them.

3 Pion and kaon electromagnetic form factor

The pion form factor in the time-like region is dominated by the ρ-meson resonance. Similarly to the pion the kaon form factor is influenced mainly by the ϕ-meson. But the contribution of ρ-ω meson mixing is also important. With the effective Lagrangian in section 2, we improve the analysis of both form factors. The pseudoscalar meson loops are also taken into account.

3.1 The ρ-meson self-energy

From the effective Lagrangian, the V-π interaction term (the 2nd term in eq. (18)) generates a vector current of pion as

$$J^\mu_\mathrm{had} = i(\pi^+\partial^\mu\pi^- - \pi^-\partial^\mu\pi^+)\,. \tag{26}$$

This coupling to the ρ-meson field produces the self-energy as shown in fig. 1, which is calculated as

$$-i\Pi^{\mu\nu} = g^2_{\rho\pi\pi} \times \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{(2p-q)^\mu(2p-q)^\nu}{(p^2-m_\pi^2+i\epsilon)((p-q)^2-m_\pi^2+i\epsilon)}\,. \tag{27}$$

The ρ-meson coupling to a conserved current implies that the self-energy is transverse, *i.e.*,

$$q_\mu\Pi^{\mu\nu}(q) = q_\nu\Pi^{\mu\nu}(q) = 0. \tag{28}$$

This property, combined with Lorentz invariance, uniquely determines the tensor structure of the self-energy

$$\Pi^{\mu\nu} = (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2})\Pi(q^2). \tag{29}$$

The full propagator of the ρ-meson is then given as

$$D^{\mu\nu} = \frac{1}{q^2-\mathring{m}_\rho^2-\Pi_\rho}(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}) + \frac{1}{\mathring{m}_\rho^2}\frac{q^\mu q^\nu}{q^2}\,. \tag{30}$$

Here, the bare ρ-meson mass, $\mathring{m}_\rho$, is introduced so that its physical mass is given by

$$m^2_{\rho\mathrm{P}} = \mathring{m}^2_\rho + \mathrm{Re}[\Pi_\rho(q^2=m_\rho^2)]. \tag{31}$$

Since the full propagator of the ρ-meson is given by eq. (30), the $\rho\to\pi\pi$ decay width at resonance is given as

$$\Gamma_{\rho\to\pi\pi} = -\mathrm{Im}\Pi_\rho(q^2=m_\rho^2)/m_\rho. \tag{32}$$

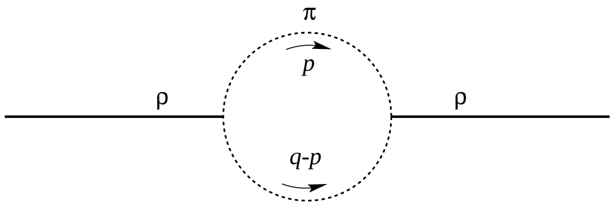

Fig. 1. ρ-meson self-energy.

3.2 Regularization

To calculate the self-energy of fig. 1, we use a Pauli-Villars regularization method [10]. The regularized self-energy is given as

$$\Pi^{\mu\nu}(q) = \tilde{\Pi}^{\mu\nu}(q,m_\pi) - \sum B_i\tilde{\Pi}^{\mu\nu}(q,\Lambda_i)\,. \tag{33}$$

Once the fictitious higher masses Λ_i are fixed, the coefficients B_i are determined by requiring that the self-energy be finite. Since this term, in its unregularized form, is quadratically divergent, we need two subtractions and obtain

$$B_1 = \frac{\Lambda_2^2-m_\pi^2}{\Lambda_2^2-\Lambda_1^2}\ ,\ B_2 = \frac{\Lambda_1^2-m_\pi^2}{\Lambda_1^2-\Lambda_2^2}. \tag{34}$$

Then, from the conditions that Λ_2 goes to infinity and Λ_1 is fixed to 1 GeV, the regularized self-energy is obtained as follows:

$$\mathrm{Re}[\Pi] = -\frac{g^2_{\rho\pi\pi}}{24\pi^2}q^2 \times\left(\mathcal{G}(q,m_\pi)-\mathcal{G}(q,\Lambda_1)+4(\Lambda_1^2-m_\pi^2)/q^2+\ln(\frac{\Lambda_1}{\mathrm{m}_\pi})\right),$$
$$\mathrm{Im}[\Pi] = -\frac{g^2_{\rho\pi\pi}}{48\pi}q^2 \times\left((1-\frac{4m_\pi^2}{q^2})^{3/2}\Theta(q^2-4m_\pi^2)-(1-\frac{4\Lambda_1^2}{q^2})^{3/2}\Theta(q^2-4\Lambda_1^2)\right), \tag{35}$$

where

$$\mathcal{G} = (\frac{4m_\pi^2}{q^2}-1)^{3/2}\arctan(\sqrt{\frac{4m_\pi^2}{q^2}-1}^{\,-1}),\quad 0<q^2<4m_\pi^2$$
$$= -\frac{1}{2}(1-\frac{4m_\pi^2}{q^2})^{3/2}\ln\frac{\sqrt{\frac{4m_\pi^2}{q^2}-1}+1}{\sqrt{\frac{4m_\pi^2}{q^2}-1}-1},\quad 4m_\pi^2<q^2,\quad q^2<0.$$

3.3 Renormalization

We consider a ρ-meson renormalization process. Expanding the transverse part of the ρ-meson propagator around

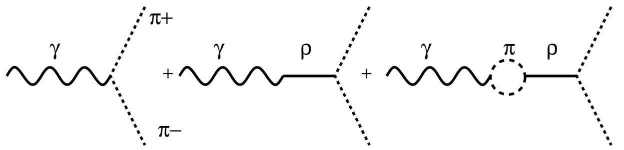

Fig. 2. Feynmann diagram of pion form factor with self-energy.

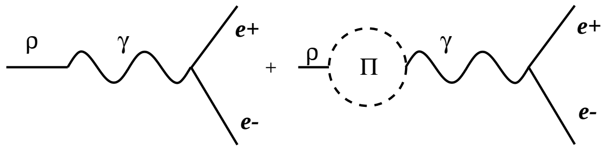

Fig. 3. ρ-meson decay.

the physical mass m_ρ, we have

$$\frac{1}{q^2 - \dot{m}_\rho^2 - \Pi_\rho(q^2)} \approx \frac{Z}{(q^2 - m_\rho^2) - iZ{\rm Im}\Pi_\rho(q^2)}, \tag{36}$$

where Z is a renormalization constant given by

$$Z = \left(1 - \frac{\rm d}{{\rm d}q^2}{\rm Re}\Pi_\rho(q^2)\mid_{(q^2=m_\rho^2)}\right)^{-1}. \tag{37}$$

Here we introduce a bare coupling constant $\dot{g}_{\rho\pi\pi}$ which is related to the physical coupling constant $g_{\rho\pi\pi}$ by $\dot{g}_{\rho\pi\pi} = Z^{1/2}g_{\rho\pi\pi}$. With the condition $Z = 1$, which means that the real part of the self-energy is completely absorbed by shifting the bare mass $\dot{m}_\rho$ to its physical value m_ρ with the coupling constant $g_{\rho\pi\pi}$ left unaltered, we can determine a new condition

$$\frac{\rm d}{{\rm d}q^2}{\rm Re}\Pi_\rho\mid_{(q^2=m_\rho^2)} = 0. \tag{38}$$

In order to satisfy this condition, we need to add an arbitrary constant term $c_\pi q^2$ in the real part of the self-energy, where c_π is given by

$$c_\pi = \frac{5.774(g - Gr^2)^2}{g^4}. \tag{39}$$

3.4 Pion electromagnetic form factor

The electromagnetic pion form factor is defined by the following matrix element:

$$\langle \pi^\pm(k')|J_\mu^{\rm em}(0)|\pi^\pm(k)\rangle = \pm(k+k')_\mu F_\pi(q^2). \tag{40}$$

The leading term of $F_\pi(q^2)$ obtained from $\mathcal{L}_{\gamma\pi}$, $\mathcal{L}_{\gamma V}$, (3rd and 5th terms, respectively) in the Lagrangian, is expressed in the following way:

$$F_\pi^{(o)}(q^2) = 1 - \frac{g_{\rho\pi\pi}}{g}\frac{q^2}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho}, \tag{41}$$

where g is a bare coupling constant which does not consider the loop effect. m_ρ and $\dot{m}_\rho$ are means of ρ-meson and bare ρ-meson, respectively. The relation of both masses is given by eq. (31).

Introducing the $\rho\pi\pi$ self-energy in fig. 2, we obtained

$$\begin{aligned} F_\pi(q^2) &= 1 - \frac{g_{\rho\pi\pi}}{g}\frac{q^2}{q^2 - \dot{m}_\rho^2 - \Pi_\rho} + \frac{\Pi_\rho}{q^2 - \dot{m}_\rho^2 - \Pi_\rho} \\ &= 1 - \frac{g_{\rho\pi\pi}}{g(q^2)}\frac{q^2}{q^2 - \dot{m}_\rho^2 - \Pi_\rho}. \end{aligned} \tag{42}$$

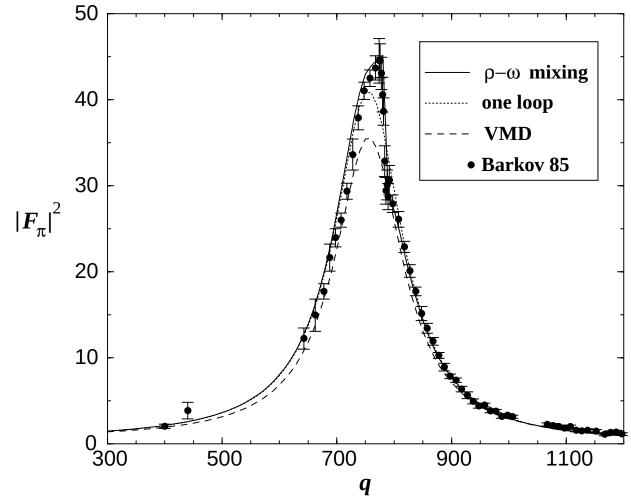

Fig. 4. Pion electromagnetic form factor in the time-like region: solid, dotted and dashed lines represent eqs.(45), (42) and (41), respectively. Here q means $\sqrt{q^2}$.

Note that not only the ρ-meson propagator, but also γ-ρ coupling is modified by the pion loop as shown in fig. 3 as follows:

$$-\frac{eq^2}{g(q^2)} = -\frac{eq^2}{g} + \frac{e\Pi_\rho}{g_{\rho\pi\pi}}. \tag{43}$$

The constant g determined from the experimental $\rho \to e^+e^-$ decay width should be compared with ${\rm Re}[g(q^2)]_{q^2=m_\rho^2}$. Using ${\rm Re}[g(q^2)]_{q^2=m_\rho^2}$ and experimental results of $\Gamma_{\rho\to\pi\pi}$, we find g and $\beta = Gr^2$ values. When g is 5.36 and β is 0.32, $g_{\rho\pi\pi}$ goes to 6.037. Under universality ($g_{\rho\pi\pi} = g_{\rho\gamma}$) used in the VMD model, the prediction of the pion form factor is underestimated compared to the experimental values. Brown *et al.* [13] allow its violation, *i.e.* $g_{\rho\pi\pi}/g_{\rho\gamma} = 1.2$ by considering the intrinsic size due to the vector meson. In the papers, for example of Brown [13] and Klingl [12], this contribution is attributed to those of vector mesons, which are incorporated by the gauge fields in the hidden gauge symmetry approach, while in this paper these fields are introduced explicitly using the nonlinear realization of the chiral symmetry exploited originally by Weinberg.

Using eqs. (31) and (32), we also find physical mass and decay width:

$$m_{\rho {\rm P}} \approx 771\,{\rm MeV},\ \Gamma_{\rho\to\pi\pi} \approx 150\,{\rm MeV},\ \dot{m}_\rho \approx 808\,{\rm MeV}. \tag{44}$$

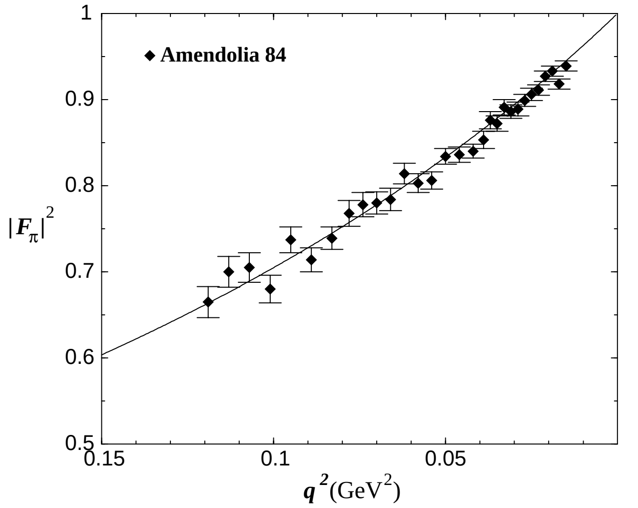

Fig. 5. Pion electromagnetic form factor in the space-like region.

Finally, the inclusion of ρ-ω mixing turned out to give another factor to eq. (42) in the following way:

$$F_\pi(q^2) = (1 - \frac{g_{\rho\pi\pi}}{g(q^2)} \frac{q^2}{q^2 - \grave{m}_\rho^2 - \Pi_\rho}) \times(1 + \frac{g(q^2)}{g_\omega} \frac{z_{\rho\omega}}{q^2 - m_\omega^2 - im_\omega\Gamma_\omega}). \qquad (45)$$

The ω-meson width $\Gamma_\omega = 8.4\,\mathrm{MeV}$ is used and the mixing parameter $z_{\rho\omega} = -4.52 \times 10^{-3}\mathrm{GeV}^2$ from ref. [12] is also exploited. The corresponding optimal result for $F_\pi(q^2)$ compared with experimental data [15] is shown in fig. 4. The dashed line plots eq. (41), which corresponds to the VMD model prediction. The dotted line plots eq. (42), *i.e.* one-loop correction is included, while the solid line represents eq. (45) in which the ρ-ω mixing contribution is taken into account.

The form factor in the space-like region ($q^2 < 0$) is also given in fig. 5. Our approach gives a good agreement with the experimental result [14]. The squared pion charged radius becomes

$$\begin{aligned}\langle r_\pi^2\rangle &= 6\frac{\mathrm{d}F_\pi}{\mathrm{d}q^2}|_{q^2=0} \\ &= \frac{6}{\grave{m}_\rho^2}(\frac{g_{\rho\pi^+\pi^-}}{g} - c_\pi + \frac{g^2_{\rho\pi^+\pi^-}}{24\pi^2}\ln(\frac{\Lambda_1}{m_\pi})) \\ &= 0.447\,\mathrm{fm}^2. \end{aligned} \qquad (46)$$

Using the constants determined before, eq. (46) yields a good agreement with the experimental value $\langle r_\pi^2\rangle = (0.44 \pm 0.01)\,\mathrm{fm}^2$. From KSRF relation $\grave{m}_\rho^2 = 2g^2_{\rho\pi\pi}f_\pi^2$, the mean square radius of the pion becomes

$$\langle r_\pi^2\rangle = (\frac{1}{4\pi f_\pi})^2 \ln(\frac{\Lambda_1^2}{m_\pi^2}) + \mathrm{const}\ . \qquad (47)$$

Therefore, in the chiral limit ($m_\pi \to 0$), the pion radius diverges logarithmically, which is consistent with ChPT [12].

3.5 Kaon electromagnetic form factor

The electromagnetic form factor of a charged kaon is also defined by

$$\langle K^\pm(k')|J_\mu^{\mathrm{em}}(0)|K^\pm(k)\rangle = \pm(k+k')_\mu F_K(q^2). \qquad (48)$$

The leading behavior of $F_K(q^2)$ is obtained just by transcribing the previous formalism developed for $F_\pi(q^2)$ in eq. (42) and replacing the ρ-meson by the ϕ-meson and the pion loop by the kaon loop. It leads to yield the following result:

$$\begin{aligned}F_K(q^2) &= 1 + \frac{\sqrt{2}}{3}\frac{g_{\phi K^+K^-}}{g}\frac{q^2}{q^2 - \grave{m}_\phi^2 - \Pi_\phi} + \frac{\Pi_{\phi\to K^+K^-}}{q^2 - \grave{m}_\phi^2 - \Pi_\phi} \\ &= 1 + \frac{\sqrt{2}}{3}\frac{g_{\phi K^+K^-}}{g(q^2)}\frac{q^2}{q^2 - \grave{m}_\phi^2 - \Pi_\phi}\ , \end{aligned} \qquad (49)$$

where the ϕ-meson self-energy has the contributions not only from K^+K^- but also from $K_L^0 K_S^0$, *i.e.*,

$$\Pi_\phi = \Pi_{\phi\to K^+K^-} + \Pi_{\phi\to K_L^0 K_S^0}\ . \qquad (50)$$

The photon coupling of the ϕ-meson is also modified by means of the charged kaon loop including the renormalization

$$\frac{1}{g_\phi(q^2)} = \frac{1}{g} + \frac{3}{\sqrt{2}}\frac{\Pi_{\phi\to K^+K^-}}{g_{\phi K^+K^-}q^2}. \qquad (51)$$

Considering the additional contributions of both ρ-meson and ω-meson, we obtain the final form of the charged kaon form factor

$$\begin{aligned}F_K(q^2) = 1 &+ \frac{\sqrt{2}}{3}\frac{g_{\phi K^+K^-}}{g_\phi(q^2)}\frac{q^2}{q^2 - \grave{m}_\phi^2 - \Pi_\phi} \\ &- \frac{g_{\rho K^+K^-}}{g_\rho(q^2)}\frac{q^2}{q^2 - \grave{m}_\rho^2 - \Pi_\rho} \\ &- \frac{4}{3}\frac{g_{\omega K^+K^-}}{g_\omega(q^2)}\frac{q^2}{q^2 - \grave{m}_\omega^2 - \Pi_\omega}. \end{aligned} \qquad (52)$$

Figure 6 shows the charged kaon form factor at the time-like region compared with experimental data [16]. Here the coupling constant $\mathrm{Re}[g_\phi(q^2)]$ is approximately 6.5; and the physical ϕ-meson mass and the $\phi \to K^+K^-$, $\phi \to K_S^0 K_L^0$ decay widths are given by

$$m_{\phi\mathrm{P}} \approx 1019\,\mathrm{MeV},\ \grave{m}_\phi \approx 940\,\mathrm{MeV},$$
$$\Gamma_{\phi\to K^+K^-} \approx 2.32\,\mathrm{MeV},\ \Gamma_{\phi\to K_S^0K_L^0} \approx 1.517\,\mathrm{MeV}.$$

The kaon form factor in the space-like region $q^2 < 0$ is shown in fig. 7. The final inclusion of the ρ- and ω-meson contributions successfully reproduces the experimental results [17]. The calculation of the mean square radius of the charged kaon is also successfully performed with the

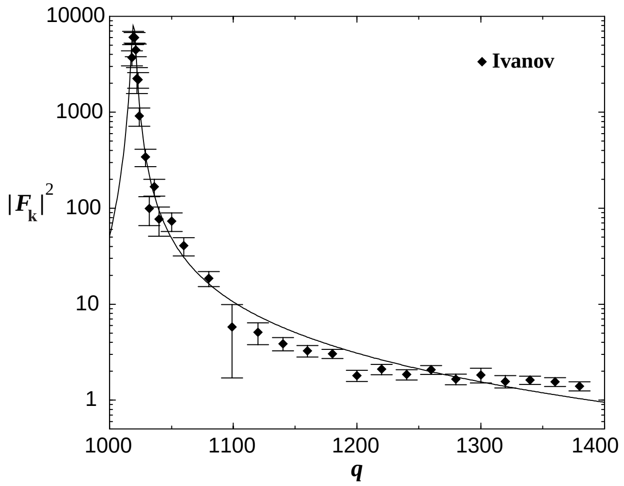

Fig. 6. Kaon electromagnetic form factor in the time-like region. Here q means $\sqrt{q^2}$.

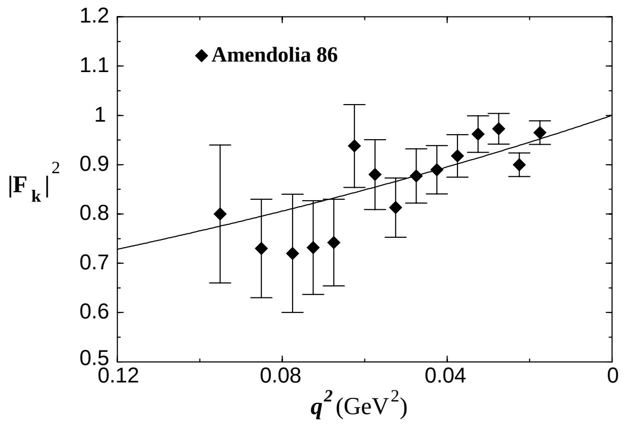

Fig. 7. Kaon electromagnetic form factor in the space-like region.

ρ- and ω-meson contribution

$$
\begin{aligned}
\langle r_{K^\pm}\rangle &= 6\frac{\mathrm{d}F_K}{\mathrm{d}q^2}|_{q^2=0}\\
&= \frac{6}{\dot m_\rho^2}\Big(\frac{g_{\rho K^+K^-}}{g} - c_{\rho K} + \frac{g^2_{\rho K^+K^-}}{24\pi^2}\ln\Big(\frac{\Lambda_1}{m_K}\Big)\Big)\\
&+ \frac{6}{\dot m_\omega^2}\Big(\frac{4}{3}\frac{g_{\omega K^+K^-}}{g} - c_{\omega K} + \frac{g^2_{\omega K^+K^-}}{24\pi^2}\ln\Big(\frac{\Lambda_1}{m_K}\Big)\Big)\\
&- \frac{6}{\dot m_\phi^2}\Big(\frac{\sqrt{2}}{3}\frac{g_{\phi K^+K^-}}{g} + c_{\phi K} - \frac{g^2_{\phi K^+K^-}}{24\pi^2}\ln\Big(\frac{\Lambda_1}{m_K}\Big)\Big)\\
&= 0.332\,\mathrm{fm}^2\,, \qquad (53)
\end{aligned}
$$

where $c_{\rho K}, c_{\omega K}$ and $c_{\phi K}$ determined from the renormalization condition eq. (38) are $c_{\rho K} = 0.0308$, $c_{\omega K} = 0.0175$ and $c_{\phi K} = 0.171$. This value agrees well with the experimental value [17]

$$\langle r^2_{K_\pm}\rangle = (0.34 \pm 0.05)\,\mathrm{fm}^2. \qquad (54)$$

As in the pion case, in the chiral limit, the mean squared radius of the charged kaon also diverges logarithmically.

Finally, the neutral kaon mean square radius is explored. It is obtained from eq. (53) replacing the K^+K^- coupling by $K^0_L K^0_S$. The sign for the ρ-meson contribution is changed as $g_{\rho K^0_L K^0_S} = -g_{\rho K^+K^-} = 0.201$, while the ϕ-meson contribution remains as $g_{\phi K^0_L K^0_S} = g_{\phi K^+K^-} = -4.72$. But, from nonet mixing, the ω-meson contribution in the case of neutral kaon is different from that of the charged kaon as follows:

$$
\begin{aligned}
g_{\omega K^+K^-} &= \frac{(m^2_\phi + m^2_\omega)}{4gf^2_K}\Big(1 - \frac{\beta_{\omega K}}{g}\Big) = 1.51\,,\\
g_{\omega K^0_L K^0_S} &= \frac{(m^2_\phi - m^2_\omega)}{4gf^2_K}\Big(1 - \frac{\beta_{\omega K}}{g}\Big) = 0.17\,. \qquad (55)
\end{aligned}
$$

These constants play an important role in understanding the mean square radii of both neutral and charged kaon. The final form of the mean square radius of the neutral kaon is given as

$$
\begin{aligned}
\langle r_{K^0}\rangle &= 6\frac{\mathrm{d}F_K}{\mathrm{d}q^2}|_{q^2=0}\\
&= -\frac{6}{\dot m_\rho^2}\Big(\frac{g_{\rho K^0_L K^0_S}}{g} + c_{\rho K^0} - \frac{g_{\rho K^0_L K^0_S}}{24\pi^2}\ln\Big(\frac{\Lambda_1}{m_K}\Big)\Big)\\
&+ \frac{6}{\dot m_\omega^2}\Big(\frac{4}{3}\frac{g_{\omega K^0_L K^0_S}}{g} - c_{\omega K^0} + \frac{g_{\omega K^0_L K^0_S}}{24\pi^2}\ln\Big(\frac{\Lambda_1}{m_K}\Big)\Big)\\
&- \frac{6}{\dot m_\phi^2}\Big(\frac{\sqrt{2}}{3}\frac{g_{\phi K^0_L K^0_S}}{g} + c_{\phi K^0} - \frac{g^2_{\phi K^0_L K^0_S}}{24\pi^2}\ln\Big(\frac{\Lambda_1}{m_K}\Big)\Big)\\
&= -0.0549\,\mathrm{fm}^2\ . \qquad (56)
\end{aligned}
$$

The different role of the intermediate ω-meson contribution to K^0 perfectly reproduces the empirical value [19]

$$\langle r^2_{K^0}\rangle = (-0.054 \pm 0.0026)\,\mathrm{fm}^2\ . \qquad (57)$$

4 Conclusion

We extended a chiral effective Lagrangian by including the vector and the axial-vector mesons as well as pions to $SU_{\rm R}(3)\otimes SU_{\rm L}(3)$. The meson fields are introduced through the non-linear realization of chiral symmetry, which provides an easy way of imposing consistency with the ChPT. In order to have mass splitting of strange and non-strange particles, the interactions between scalar mesons and each meson, *i.e*, vector, axial-vector, and pseudoscalar mesons, are taken into account.

For the phenomenological side of this Lagrangian, pion and kaon electromagnetic form-factors and some related decays are calculated. In the process of calculating, our effective Lagrangian is shown to give a good agreement with experimental data without considering the effects from the higher orders in other effective theories.

This work was supported in part by the Korea Research Foundation.

References

1. M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. **164**, 217 (1988).
2. U.G. Meissner, Phys. Rep. **161**, 213 (1988).
3. M.C. Birse, Z. Phys. A **355**, 231 (1996).
4. T.S. Park, B.S. Han, M.K. Cheoun and S.H. Lee, J. Kor. Phys. Soc., **29**, S381 (1996).
5. J. Gasser, H. Leutwyler, Nucl. Phys. B **250**, 517 (1985).
6. G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321**, 311 (1989).
7. M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. B **259**, 493 (1985).
8. H.B. O'Connell et. al, Prog. Nucl. Part. Phys **39**, 201 (1997).
9. R.K. Bhaduri, *Models of Nucleon*, (Addison-Wesley Publishing Company, Inc., Redwood city, 1988)
10. M. Herrmann, B.L. Friman and W. Norenberg, Nucl. Phys. A **560**, 411 (1993).
11. Fayyazuddin and Riazuddin, Phys. Rev. D **36**, 2768 (1987).
12. F. Klingl, N. Kaiser and W. Weise, Z. Phys A **356**, 193 (1996).
13. G.E. Brown, M. Rho and W. Weise, Nucl. Phys A **454**, 669 (1986).
14. S.R Amendolia et al., Phys. Lett. B **146**, 116 (1984).
15. L.M. Barkov et al., Nucl. Phys. B **259**, 365 (1985).
16. P.M. Ivanov, Phys. Lett. B **107**, 297 (1981).
17. S.R Amendolia et al., Phys. Lett. B **178**, 435 (1986).
18. L.S. Celenza, X.D. Li and C.M. Shakin, Phys. Rev. C **56**, 3326 (1997).
19. W. Molzon et al., Phys. Rev. Lett. **41**, 1213 (1978).